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ORIGINAL

A proposal for an instructional methodological class on iterative numerical methods for Systems of Linear Equations

Propuesta de clase metodológica instructiva sobre métodos numéricos iterativos para Sistemas de Ecuaciones Lineales

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ABSTRACT

Introduction: the methodological work aimed to enhance teachers' pedagogical preparation to optimize the teaching-learning process, focusing on improving Curriculum E of Computer Science, specifically in the Numerical Mathematics course.
Method: an instructional methodological class was designed based on previous research conducted at the Faculty of Mathematics and Computing at the University of Havana, addressing the numerical solution of systems of linear equations through iterative methods. Documentary analysis, observation, and inductive-deductive methods were used, integrating theoretical foundations, efficient algorithms, and practical exercises. The class followed an introduction, development, and conclusions structure.
Results: the proposal combined a theoretical summary, practical exercises, and comparative approaches, providing a structured pedagogical resource to facilitate conceptual understanding and the application of numerical techniques.
Conclusions: the initiative offers an effective methodological guide for teachers, strengthening the teaching of Numerical Mathematics and promoting significant transformations in the educational process.

Keywords: Computer Science; Instructional Methodological Class; Numerical Mathematics; Methodological Work; Teaching-Educational Process.

RESUMEN

Introducción: el trabajo metodológico buscó mejorar la preparación pedagógica de los docentes para optimizar el proceso de enseñanza-aprendizaje, centrándose en el perfeccionamiento del plan de estudios E de Ciencia de la Computación, específicamente en la asignatura de Matemática Numérica.
Método: se diseñó una clase metodológica instructiva basada en investigaciones previas realizadas en la Facultad de Matemática y Computación de la Universidad de La Habana, enfocada en la solución numérica de sistemas de ecuaciones lineales mediante métodos iterativos. Se utilizaron análisis documental, observación y métodos inductivo-deductivos, la propuesta integró fundamentos teóricos, algoritmos eficientes y ejercicios prácticos. La clase siguió una estructura de introducción, desarrollo, conclusiones.
Resultados: la propuesta integró un resumen teórico, ejercicios prácticos y enfoques comparativos, proporcionando un recurso pedagógico estructurado para facilitar la comprensión conceptual y la aplicación de técnicas numéricas.
Conclusiones: la iniciativa ofrece una guía metodológica efectiva para docentes, fortaleciendo la enseñanza de la Matemática Numérica y promoviendo transformaciones significativas en el proceso educativo.

Palabras clave: Ciencia de la Computación; Clase Metodológica Instructiva; Matemática Numérica; Proceso Docente-Educativo; Trabajo Metodológico.

INTRODUCTION

Computer science is dedicated to the analysis of the theoretical principles that govern information and computational processes, as well as their application in the creation of computer systems. This field advances in parallel with the rapid development of science and technology, providing solutions to the computerization needs of today's society.[1]

According to the E curriculum,[2] the work of the Bachelor of Computer Science focuses on the creation of computational systems, using mathematical-computational approaches to solve specific or interdisciplinary problems.

The Applied Mathematics discipline in this degree program contributes to understanding the importance of applying mathematical methods and computing. It covers three main areas: Numerical Mathematics, Probability and Statistics, and Optimization. This discipline allows students to solve basic mathematics problems that cannot be addressed analytically, laying the foundation for incorporating more complex models and tools in other areas of the degree program.

In particular, Numerical Mathematics studies fundamental numerical methods for approximating mathematical solutions efficiently and algorithmically, responding to the growing need for algorithms for scientific and technical models.

The course prepares students to analyze, apply, modify, and adapt general numerical methods to specific situations. Students develop skills to use these methods efficiently in computational environments and create new algorithms that solve practical problems formulated mathematically. The content includes direct methods for systems of linear algebraic equations, matrix factorizations, and matrices with particular properties. Among the expected skills is the ability to select appropriate algorithms and adapt them as needed.

Methodological work is a set of ongoing activities carried out by teachers at all educational levels. Its objective is to improve the pedagogical-methodological and scientific preparation of educators, thus ensuring the efficient execution of the educational process. Along with other forms of professional and postgraduate development, this approach seeks to achieve teaching staff competence and generate positive influences on the comprehensive training of students. This involves setting clear objectives, organizing appropriate methods, regulating operational actions, and monitoring progress toward established goals.

Careful planning of the teaching process ensures effective teaching based on didactic principles that promote optimal learning. The teacher plays a key role in ensuring quality teaching, guiding, evaluating, and monitoring students to facilitate their comprehensive education.[3,5] Teaching methods and forms are manifested in the teaching process.[6] By the former, we mean the way in which teachers and students carry out actions to achieve objectives, and by teaching form, we mean the organizational structure that is adopted at a given moment in the teaching process in order to achieve objectives.

In this context, methodological classes have two modalities: the demonstrative methodological class (DMC) and the instructional methodological class (IMC). The IMC faces greater challenges due to its complexity. Unlike the DMC, it focuses on identified methodological problems rather than teaching a class from the program. Its essential objective is to instruct teachers in certain aspects that are at the heart of the difficulties detected.[7,8] The methodological treatment must be linked to the conceptual content of the subject or scientific aspect addressed. This involves identifying didactic contradictions between the content and its methodological orientation in order to optimize student learning.[7,9]

CMIs do not exist in isolation; they are integrated with other forms of methodological work conceived as a system from planning to execution. These classes identify priorities according to deficiencies detected through counseling and organizational control. They propose didactic solutions to the deficiencies detected. The objective of this work is to design a CMI on the numerical solution of linear equation systems (SEL) using iterative methods, serving as a guide for less experienced teachers in their development as university professors.

METHOD

Research was conducted in the field of mathematical-computational education at the Faculty of Mathematics and Computing of the University of Havana to present an instructional methodological class on the numerical solution of linear equation systems using iterative methods.

The instructional methodological objective and the methodological line of work were defined. A summary of the teaching activity to be developed was proposed, as well as the development of the class with its three parts: introduction, development, and conclusions.

DEVELOPMENT

The following section addresses the elements of the CMI according to Alonso Berenguer et al.[7]

Instructional Methodological Class

Methodological Objective

To instruct teachers on how to use methodological alternatives to develop teaching activities on the numerical solution of linear equation systems using iterative methods, in order to promote meaningful learning.

Methodological line of work

Improvement of learning management to stimulate developmental and meaningful learning in the numerical solution of systems of linear equations using iterative methods.

Degree Program: Computer Science

Discipline: Applied Mathematics.

Subject: Numerical Mathematics. Second Year.

Topic III: numerical solution of linear equation systems

Teaching Activity: iterative methods for the numerical solution of systems of linear equations.

Summary: need for iterative methods for a system of linear equations. Solving a system of linear equations using fixed-point iteration. Convergence. Jacobi iteration and its convergence conditions. Gauss-Seidel iteration and its convergence conditions. Convergence theorems. Examples. Advantages and disadvantages compared to direct methods.

Teaching method: Lecture.

Duration: 2 hours/class.

Basic bibliography:

- Burden et al.[10]. Numerical analysis. Cengage Learning, pp. 431-495.

Supplementary bibliography:

- Heath[11]. Scientific Computing: An Introductory Survey (Revised Ed). Society for Industrial and Applied Mathematics, pp. 466-470.

Methods: expository, collaborative, and participatory.

Resources: whiteboard and markers, slides, computers.

Assessment: observation of student work, oral questions, and group discussions.

The teacher should set an example with their punctuality, demeanor, and appearance. The classroom should be clean, well-lit, and well-ventilated, with no objects that could distract students.

Introduction

Student attendance and punctuality are checked. Next, a comment is made to motivate the class, explaining that many problems in applied mathematics can be reduced to problems involving the solution of linear systems of equations. For example, the solution of ordinary or partial differential equations using finite difference methods, eigenvalue problems, data fitting using least squares, and polynomial approximation; the latter two topics will be covered later in the course.

As this is a new topic, a reminder is given of a previous lecture that addressed the topic of error analysis in the Gauss elimination method. Along with this, control questions are asked to obtain feedback on the previous lecture, encouraging debate with the students and evaluating those who voluntarily decide to answer, according to the quality of their answers, giving them a grade (2, 3, 4, or 5).

To do this, the definition of the problem to be solved is recalled:$Ax=b$,A is square and invertible. Students are familiar with this definition from the Algebra I course, which is taught in the first year of the degree program. At this point, students are asked: What does it mean to solve a SEL? What methods of solving SEL do you know? This is so that they can link the content with the previous course, where they solved SEL analytically and manually, with the current one.

It is pointed out that the use of determinants for solving SELs involves a high computational cost, specifically in higher-order matrices. Taking advantage of this, students are asked: What is the computational cost of solving a SEL using Cramer's rule and expansion by minors? This form of solution requires approximately $3(n+1)!$ operations, which is computationally impractical.

Then, they are asked: What is a direct method? They are reminded that a direct method for solving an SEL is one that, in the absence of rounding errors, will produce the exact solution after a finite number of elementary arithmetic operations. It is emphasized that, in practice, because these errors exist, direct methods do not lead to exact solutions. This third topic of the course is thus related to the first, which refers to floating-point arithmetic and error propagation. It is also pointed out that in direct methods there is no truncation error and

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that the fundamental direct method is the Gauss elimination method, which they have already learned about in the lecture.

In addition, it is pointed out that the solution of large and dense SELs is computationally impractical; the calculation of numerical determinants, although not advisable for solving systems, is sometimes required for other purposes; matrix inversion is an inefficient procedure for a system of ordern>4 ; and there are ill-conditioned problems, where the condition of the matrix involved allows error propagation when using the Gauss elimination algorithm. All of this led to the study of iterative methods for solving SELs.

It is explained that iterative methods produce an infinite sequence of approximate solutions, a sequence that, under certain conditions, converges to the exact solution of the problem.

These iterative methods are used in sparse matrices, where the memory requirements are (n) and not (n^2). Sometimes, the matrix has certain properties that allow iterative methods to be used successfully. After looking at direct methods, the study of iterative methods for solving an SEL is discussed.

Following this presentation, the summary of the conference is presented and its objectives are outlined, as well as the basic and complementary bibliography for further study of the content.

Objectives

- Describe the characteristics of direct methods and iterative methods for solving systems of linear equations.
- Describe Jacobi's and Seidel's iterative methods for a system of linear equations and their convergence conditions.
- Solve a system of linear equations using Jacobi and Seidel algorithms manually or computationally.
- Perform a comparative analysis between the Jacobi and Seidel algorithms.
- Decide on the preferred algorithms for solving a system of linear equations.

Development of the lecture

Subsequently, the content is organized and the lecture is delivered, applying the aforementioned methods and using the necessary resources. Student progress is monitored and any clarifications deemed necessary are provided at any given time.

It is explained that the iterative methods for solving simpler and better-known linear systems are fixed-point iterations. The fixed-point iteration for a nonlinear equation taught in topic II of the Numerical Mathematics course is reviewed:

$$f(x) = 0 \rightsquigarrow x = g(x),$$
$$x_{k+1} = g(x_k).$$

The fixed-point iteration studied can be generalized to a system of nonlinear equations:

$$f(x) = 0,$$
$$f: S \to \mathbb{R}^n,\ S \subseteq \mathbb{R}^n,$$
$$f(x) = 0 \rightsquigarrow x = g(x),$$
$$x^{(k+1)} = g\left(x^{(k)}\right),\ k = 0,\ 1,\ 2, \cdots$$

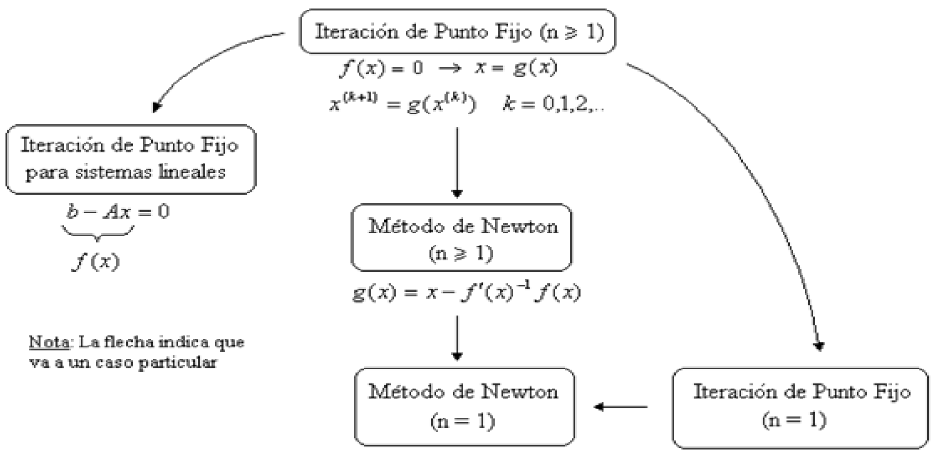

Figure 1. Fixed-point iteration approaches for solving nonlinear equations and systems of linear equations

Figure 1 shows the diagram where iterative methods for systems of linear equations are located within fixed-point iteration approaches.

Next, it is shown that fixed-point iteration for linear systems is a special case of fixed-point iteration for nonlinear systems, where the following must be solved:

$$Ax = b \text{ o } f(x) = b - Ax = 0 \rightsquigarrow x = Bx + c,$$
$$x^{(k+1)} = Bx^{(k)} + c,\ k = 0,1,\cdots,$$

Where B is a matrixn×n (called the iteration matrix) andc is an vector.

This iteration is applied in the hope that it will converge to the exact solution ξ (which is unknown). Students are then asked the following question: When does a sequence of vectors converge to a vector ξ? Here, a contrast is made with fixed-point iterations for solving a nonlinear equation, and the need to define what convergence to a vector means is shown.

In this way, vector sequences are established that are obtained by successively applying iterative methods to solve a system of linear equations, which is expected to converge to the exact solution ξ:

$$\begin{matrix} x^{(0)} & x^{(1)} & x^{(2)} & \cdots \rightarrow & \xi \\ \begin{pmatrix} x_1{}^{(0)} \\ x_2{}^{(0)} \\ \vdots \\ x_n{}^{(0)} \end{pmatrix} & \begin{pmatrix} x_1{}^{(1)} \\ x_2{}^{(1)} \\ \vdots \\ x_n{}^{(1)} \end{pmatrix} & \begin{pmatrix} x_1{}^{(2)} \\ x_2{}^{(2)} \\ \vdots \\ x_n{}^{(2)} \end{pmatrix} & \begin{matrix} \cdots \rightarrow \\ \cdots \rightarrow \\ \vdots \\ \cdots \rightarrow \end{matrix} & \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{pmatrix}. \end{matrix}$$

This convergence is formally achieved when:

$$\lim_{k\to\infty} x^{(k)} = \xi \Leftrightarrow \left[\text{para } i = 1,2,\cdots,n,\ \lim_{k\to\infty} x_i{}^{(k)} = \xi_i\right],$$

Or, equivalently:

$$\lim_{k\to\infty} \left\| x^{(k)} - \xi \right\| = 0$$

The simplest case of fixed-point iteration for SEL is Jacobi iteration. Suppose that all elements of the diagonal of A are nonzero. In this iteration, B=BJ=(bij) andc=(ci) are taken as:

$$b_{ij} = \begin{cases} -\dfrac{a_{ij}}{a_{ii}}, & \text{si } i \neq j \\ 0, & \text{si } i = j \end{cases},$$
$$c_i = \frac{b_i}{a_{ii}},\ \ i,\ j = 1,\ 2,\cdots,\ n.$$

It can be seen that this is equivalent to solving for Ax=b, xi in thei th equation.

Below is an example of the construction of the Jacobi iteration matrix: given the system:

$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ 2x_1 + 10x_2 + x_3 &= 13 \\ x_1 + 3x_2 + 10x_3 &= 14 \end{aligned}$$

Whose exact solution is $\xi=(1,1,1)^T$. Fori=1,2,3, we solve xi in thei th equation:

$$\begin{aligned} x_1 &= \qquad\quad -\tfrac{1}{10}x_2 - \tfrac{1}{10}x_3 + \tfrac{12}{10} \\ x_2 &= -\tfrac{2}{10}x_1 \qquad\quad - \tfrac{1}{10}x_3 + \tfrac{13}{10}, \\ x_3 &= -\tfrac{1}{10}x_1 - \tfrac{3}{10}x_2 \qquad\quad + \tfrac{14}{10} \end{aligned}$$

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$$x = \begin{pmatrix} 0 & -0.1 & -0.1 \\ -0.2 & 0 & -0.1 \\ -0.1 & -0.3 & 0 \end{pmatrix} x + \begin{pmatrix} 1.2 \\ 1.3 \\ 1.4 \end{pmatrix}$$

$$x = B_J x + c.$$

When implementing the method, we obtain:

$$\begin{matrix} B_J & c & x^{(0)} & x^{(1)} & x^{(2)} \\ \begin{pmatrix} 0 & -0.1 & -0.1 \\ -0.2 & 0 & -0.1 \\ -0.1 & -0.3 & 0 \end{pmatrix} & \begin{pmatrix} 1.2 \\ 1.3 \\ 1.4 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 1.2 \\ 1.3 \\ 1.4 \end{pmatrix} & \begin{pmatrix} 0.93 \\ 0.92 \\ 0.89 \end{pmatrix} \end{matrix}$$

Where we can see that the iteration is converging to $\xi=(1,1,1)^T$.

Next, it is noted that the fixed-point iteration for SEL does not always converge, as happened in the example. If we take the SEL:

$$\begin{aligned} 4x_1 - 3x_2 + 5x_3 &= 25 \\ 3x_1 + 2x_2 + +x_3 &= 11, \\ 4x_1 - 2x_2 + 3x_3 &= -4 \end{aligned}$$

The Jacobi method does not converge to the solution of the system, which is $\xi=(3,-1,2)^T$. Table 1 illustrates this behavior. Students are encouraged to implement this algorithm in the Python programming language and replicate the examples where the iteration converges and where it does not.

Table 1. Behavior of Jacobi iteration in a case where there is no convergence

Iteration	x_1	x_2	x_3
0	0,0	0	0
1	6,25	5,5	-1,3333
2	12,0417	2,5417	10,6667
3	-8,9896	-23,2292	13,0298
4	-27,4566	5,9566	-28,8056
5	46,7244	75,4905	-33,9711
6	105,3317	30,6155	111,2928
7	-155,8277	263,7907	118,6985

That is why we will study convergence conditions, but first we need to define the spectral radius of a matrix as:

$$\rho(B) = \max_{i=1,2,\cdots,n} \{|\lambda_i| : \lambda_i \text{ es un valor propio de } B\}.$$

For its geometric interpretation, let us assume that |λ1|≥|λ_2|≥⋯≥|λn|. For example, if n=5, the spectral radius of a matrix is the radius of the smallest circle in the complex plane with center at the origin that contains all the eigenvalues of the matrix (figure 2).

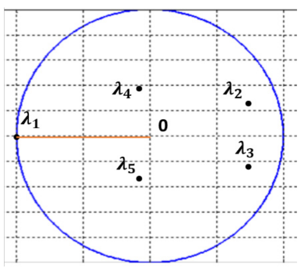

Figure 2. Geometric interpretation of the spectral radius of a matrix of order 5

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The spectral norm and the spectral radius are related:

$$\|A\|_2 = \|A\|_S = \sqrt{\rho(A^T A)}$$

Which can be justified and is presented as an independent exercise.

Once the spectral radius has been defined, we present a necessary and sufficient condition for the convergence of fixed-point iteration for a system of linear equations:

Theorem

Let Ax=b be a linear system, and $x^{(k+1)}=Bx^{(k)}+c$,k=0,1,⋯, a corresponding fixed-point iteration. This iteration is convergent (independently of $x^{(0)}$ and b) if and only if ρ(B)<1 .

For every norm, it holds that $\rho(B) \leq \|B\|$, that is, the spectral radius of a matrix is always less than or equal to any of its norms. The proof of this property is also assigned as homework.

In our example:

$$B_J = \begin{pmatrix} 0 & -0.1 & -0.1 \\ -0.2 & 0 & -0.1 \\ -0.1 & -0.3 & 0 \end{pmatrix},$$
$$\|B_J\|_1 = 0.4,$$
$$\|B_J\|_2 \approx 0.34,$$
$$\|B_J\|_\infty = 0.4,$$
$$\|B_J\|_F = \sqrt{0.17} \approx 0.412.$$

And indeed, ρ(B)≈0,29 is less than the previous values.

What are the consequences of this property and the stated theorem? This implies that the fixed-point iteration converges (independently of $x^{(0)}$ and b) if and only if ‖B‖<1, for some matrix norm. In practice, we most often apply sufficiency, that is, we try to find a matrix norm such that ‖B‖<1. This ensures convergence.

Before analyzing the convergence of the Jacobi method, the following definition is presented: A matrix A=(aij) of order n is strictly row-diagonally dominant if:

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \text{ para } i = 1,2,\cdots,n.$$

For example, the matrix:

$$A = \begin{pmatrix} -5 & 2 & -2 \\ 2 & 4 & -1 \\ -1 & 1 & -3 \end{pmatrix},$$

Is strictly row-diagonally dominant.

As a consequence, for the system Ax=b, if is strictly row-diagonally dominant, then the corresponding Jacobi iteration is convergent (sufficient condition of the theorem). Students are encouraged to verify that in the example SEL:

$$\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ 2x_1 + 10x_2 + x_3 &= 13, \\ x_1 + 3x_2 + 10x_3 &= 14 \end{aligned}$$

The Jacobi method converges. With these elements, the Jacobi iteration algorithm is presented. Given the SEL Ax=b of ordern whose coefficient matrix A=(aij) has all diagonal elements unequal to zero.

Calculate the elements of BJ=(bij) andc=(ci) using:

$$b_{ij} = \begin{cases} -\frac{a_{ij}}{a_{ii}}, & si\ i \neq j \\ 0, & si\ i = j \end{cases}; \quad c_i = \frac{b_i}{a_{ii}},\ i,j = 1,\cdots,n$$

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Take an $x^{(0)}$, for example $x^{(0)}=0$.
For k=1,2,⋯, until finished, repeat:
For i=1,2,⋯,n , repeat:

$$x_i^{(k)} = \sum_{\substack{j=1 \\ j\neq i}}^{n} b_{ij} x_j^{(k-1)} + c_i$$

It should be noted that Jacobi iteration is also called the simultaneous displacement method, because to calculate any component of $x^{(k)}$, it only uses components of $x^{(k-1)}$ (see the initial numerical example again).

On the other hand, there is another form of iteration called Gauss-Seidel iteration (also called the successive displacement method), because to calculate a component of $x^{(k)}$, we use the previous components of the vector itself.

The Gauss-Seidel iteration for solving an SEL is shown below:

Given the linear systemAx=b of ordern whose coefficient matrix A=(aij) has all diagonal elements unequal to zero.

Calculate the elements of BJ=(bij) andc=(ci) using:

$$b_{ij} = \begin{cases} -\frac{a_{ij}}{a_{ii}}, & si\ i \neq j \\ 0, & si\ i = j \end{cases}; \ c_i = \frac{b_i}{a_{ii}} \ , \ i,j = 1,\cdots,n$$

Take an $x^{(0)}$, for example $x^{(0)}=0$.
For k=1,2,⋯, until finished, repeat:
To i=1,2,⋯,n repeat:

$$x_i^{(k)} = \sum_{j=1}^{i-1} b_{ij} x_j^{(k)} + \sum_{j=i+1}^{n} b_{ij} x_j^{(k-1)} + c_i$$

Then, a numerical example of this iteration is presented:

$$\begin{matrix} B_J & & c & x^{(0)} & x^{(1)} & x^{(2)} \end{matrix}$$
$$\begin{pmatrix} 0 & -0.1 & -0.1 \\ -0.2 & 0 & -0.1 \\ -0.1 & -0.3 & 0 \end{pmatrix} \begin{pmatrix} 1.2 \\ 1.3 \\ 1.4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1.2 \\ 1.06 \\ 0.962 \end{pmatrix} \begin{pmatrix} 0.9978 \\ 1.00424 \\ 0.998948 \end{pmatrix}.$$

It can be seen that the iteration is converging to $\xi=(1,1,1)^T$. Furthermore, it does so more quickly than in Jacobi's method. To verify this, students are asked to calculate $\|x^{(k)}-x^{(k-1)}\|_\infty$, for k=1,2 .

Next, the convergence conditions for both methods are presented: LetB_J be the Jacobi iteration matrix. It can be shown that:

1. If $\|B_J\|_\infty<1$, then both methods converge, and the Gauss-Seidel method converges faster.
2. If $\|B_J\|_1<1$, then both methods converge, but it cannot be stated in principle which one converges faster.
3. If $\|B_J\|_F<1$, then the Jacobi method converges, but nothing can be said about the convergence of the Gauss-Seidel iteration.
4. If A is strictly row-diagonally dominant, then we are in case 1.
5. If A is symmetric and positive definite, then the Gauss-Seidel method converges.

CONCLUSIONS

The content taught is systematized and the advantages and disadvantages of the methods for numerically solving an SEL are summarized.

Direct techniques theoretically provide the exact solution of the system in a finite number of steps. In practice, the solution will have rounding error due to floating point arithmetic, so the error must be controlled. Iterative techniques are almost never used to solve small linear systems, as the time required for sufficient accuracy exceeds that required for direct techniques. For large systems with a high percentage of zero entries (sparsity), iterative techniques are computationally efficient.

In addition, direct methods do not require an initial estimate of the solution. Direct methods are good for producing high accuracy, but they are not useful if only low accuracy is needed. Iterative methods depend on matrix properties and converge slowly in ill-conditioned systems. Direct methods are more robust in both senses. Iterative methods are good when the matrix is produced on demand.

Students are then asked questions about the key aspects to be learned in this class that will enable them to meet its objectives:

1) Why are iterative methods necessary to solve an SEL, if direct methods such as Gauss's already exist?
2) How is the solution of an SEL expressed as a fixed-point iteration?
3) What does it mean for this iteration to converge in the case of an SEL?
4) What iterations have we studied? Do they always converge?
5) How can convergence conditions be checked computationally?

Finally, as an independent study, the following exercise is intended to apply the convergence conditions of the Jacobi and Gauss-Seidel methods, numerically verify these conditions, and understand the advantages and disadvantages of the methods:

Consider the system Ax=b, where:

$$A=\begin{pmatrix} 3 & 0 & -1 \\ -1 & 2 & 0 \\ 1 & 0 & -2 \end{pmatrix}, b=\begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix}$$

1. Can we theoretically ensure that the Jacobi method converges?
2. Perform two steps of the method.
3. Is it converging in practice? Explain.
4. Same as (a) but with respect to the Gauss-Seidel method.
5. The same as (b) but with respect to the Gauss-Seidel method.
6. The same as (c) but with respect to the Gauss-Seidel method.
7. Compare the results of (e) and (b).
8. Let BGS, and cGS be the iteration matrix and constant vector, respectively, of the Gauss-Seidel method. Find BGS, and cGS .
9. Is ‖BGS‖<1 for any norm? How important is this?
10. Perform $x^{(k+1)}=B_{GS}\, x^{(k)}+c_{GS}$ for k=0,1 .
11. Compare the results of parts j) and e).

CONCLUSIONS

The proposed instructional methodology class seeks to train teachers in giving lectures on the numerical solution of linear equation systems (LES) using iterative methods, promoting meaningful learning in future computer science graduates. This methodological approach considers the nature of the content and the relationships between the topics, the subject, and the discipline, favoring the personal and professional development of the student.

During the lecture, the contents were summarized, group dynamics were carried out to evaluate learning, and typical exercises on the topic were solved, comparing different approaches to addressing LES. The methodology presented can serve as a model for other teachers in the teaching of Numerical Mathematics in Cuban and foreign universities.

BIBLIOGRAPHIC REFERENCES

1. Figueroa KM, Hoyos G de J, Martínez M de L, Morales R. La importancia del pensamiento computacional en la era digital. In: Carreño (eds.). Pensamiento Computacional en Iberoamérica. México: Academia Mexicana de Computación; 2023. https://amexcomp.mx/media/publicaciones/pensamiento-computacional-version-final-portada-reduced.pdf#page=34

2. Universidad de La Habana. Plan de Estudio "E", Carrera Ciencia de la Computación. La Habana, Cuba: 2017.

3. García González MC, Varela de Moya HS, Espíndola Artola A. Las formas del trabajo docente metodológico en el contexto actual de la educación superior. Humanidades Médicas 2019;19(3):607-36. http://scielo.sld.cu/scielo.php?script=sci_abstract&pid=S1727-81202019000300607&lng=es&nrm=iso&tlng=es

4. Ministerio de Educación Superior. Resolución 47: Reglamento Trabajo Docente y Metodológico. Ministerio de Educación Superior, Cuba. La Habana, Cuba: Ministerio de Educación Superior, Cuba; 2022.

5. Chaviano O, Baldomir T, Coca O, Gutiérrez A. La evaluación del aprendizaje: nuevas tendencias y retos para el profesor. Edumecentro 2016;8(4):191–205. https://www.medigraphic.com/cgi-bin/new/resumen.cgi?IDARTICULO=68398

6. Addine F, Recarey S, Fuxá M, Fernández S. Didáctica: teoría y práctica. La Habana, Cuba: Editorial Pueblo y Educación; 2020.

7. Alonso I, Gorina A, Iglesias N. ¿Cómo estructurar y desarrollar una clase metodológica instructiva? Rev Roca 2020. https://dialnet.unirioja.es/servlet/articulo?codigo=7414322

8. Guerrero E. Análisis lógico semántico del contenido de las clases metodológicas instructivas. Didasc@ lia Didáctica y Educ 2018;9(3):129–44. https://revistas.ult.edu.cu/index.php/didascalia/article/view/778

9. Méndez AR, García O. El problema conceptual metodológico y el docente metodológico. Aproximación a sus rasgos distintivos. Roca Rev Científico-Educaciones la Prov Granma 2018;14(3):133–42. https://dialnet.unirioja.es/servlet/articulo?codigo=6759635

10. Burden RL, Faires DJ, Burden AM. Análisis numérico. Santa Fe, México: Cengage Learning Editores; 2017. https://biblioteca.uazuay.edu.ec/buscar/item/80835

11. Heath MT. Scientific Computing Scientific Computing: An Introductory Survey. Revised Ed. Philadelphia: Society for Industrial and Applied Mathematics; 2018. https://e6.ijs.si/~roman/files/tmp/M.Heath-SComputing/scientific-computing-michael-t-heath.pdf

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